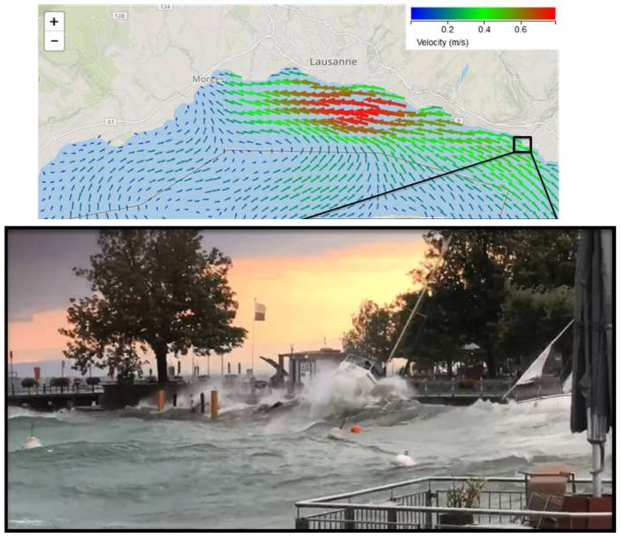
# Limnology, Chapter 7:
# Surface and bottom boundary layers in lakes




Lake Geneva, Vevey, 6th August 2018

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## Oral exam:
## Monday 23 June 2025 from 09h to 16h, GC A1 416

| | Preparation | Examen | |
|---|---|---|---|
| 1 | 09.00 - 09.15 | 09.30 - 09.45 | Tiago Gonçalves Jessica |
| 2 | 09.30 - 09.45 | 09.45 - 10.00 | Stöckli Marlen Tabea |
| 3 | 10.30 -10.45 | 10.45 -11.00 | Spoletini Nicola |
| 4 | 10.45 -11.00 | 11.00 -11.15 | Schmidt Julia |
| | | | |
| 5 | 11.00 -11.15 | 11.15 -11.30 | Noske Theresa |
| 6 | 11.15 -11.30 | 11.30 -11.45 | Istepanyan Anna |
| 7 | 11.45 - 12.00 | 12.00-12.15 | Henrioux Sven |
| 8 | 12.00-12.15 | 12.15-12.30 | Gremion Benjamin Samuel |
| | | | |
| 9 | 13.30 -14.00 | 14.00 - 14:15 | Bojaly Clua Alberto Nicolas |
| 10 | 14.00 - 14:15 | 14.15 - 14:30 | Faval Emma Charlotte Joséphine |
| 11 | 14.15 - 14:30 | 14.30 - 14:45 | Clément Julien |
| 12 | 14.30 - 14:45 | 14.45 - 15:00 | Breton Jules Louis Patrick |
| | | | |
| 13 | 15.15 -15.30 | 15.30 -15.45 | Barthez Mathias Jean |
| 14 | 15.30 -15.45 | 15.45 -16.00 | Aymon Joël |

You are allowed to bring with you a A4 page (one side)

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# Limnology (TU at 09.15)

Updates for the next classes

| class no | Date | Teacher | |
|---|---|---|---|
| 9 | 15.04.2025 | Odermatt | Remote sensing / Water quality |
| | 22.04.2025 | Holiday | Easter |
| 10 | 29.04.2025 | Tofield-Pasche | Field work on LéXPLORE platform/ Convection |
| 11 | 6.05.2025 | Tofield-Pasche | Convection / Geochemistry |
| 12 | 13.05.2025 | Tofield-Pasche | Geochemistry/Field work on LéXPLORE platform |
| 13 | 20.05.2025 | Tofield-Pasche | Summary and Energy |
| 14 | 27.05.2025 | Free | |





# Remote sensing - preparation

- download and install the open source satellite image processing toolbox from ESA, including the 'Sentinel Toolboxes' (top row): https://step.esa.int/main/download/snap-download/
- After installation, please start SNAP, choose the option 'Tools/plugins' from the menu bar at the top, and select and install 'IdePix MERIS' from the list of 'Available plugins'.
- download also at least one of the three satellite images (500 MB each, dropbox preview will show an error, but downloading works)

- All information will be on the moodle (and links to the images)
- In case of problem, daniel.odermatt@eawag.ch

# After the holidays

**LéXPLORE visit on 29 April or 13 May:**

- Confirmed on Monday 28 April or 13 May. Please also check your email in the morning, in case of last minute cancellation.
- If no visit → course in usual room
- Meeting point in Pully-Plage near the Club Nautique at 9h30 to 11h
- 1 group of 7 persons
- The visit will last 1h30 (including the boat transfers)

Please confirm your participation
https://doodle.com/meeting/participate/id/dGWRk4rd

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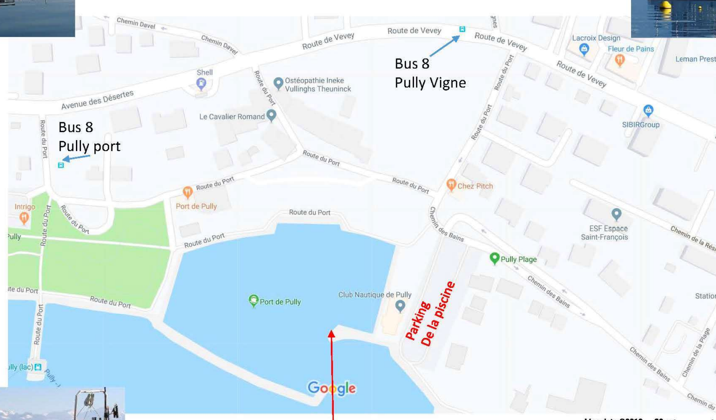
# Meeting point

**Accès à la plateforme LéXPLORE**

**Depuis le débarcadère proche du Club Nautique de Pully**




Lieu d'embarquement sur le bateau "la Seiche"

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# Learning objectives

**Chapter 7: Surface and bottom boundary layers**

Today you will learn about :

1. Wind forcing (SBL)
2. Friction (BBL)
3. LOG-layer / turbulence
4. Viscous boundary layer
5. Diffusive boundary layer

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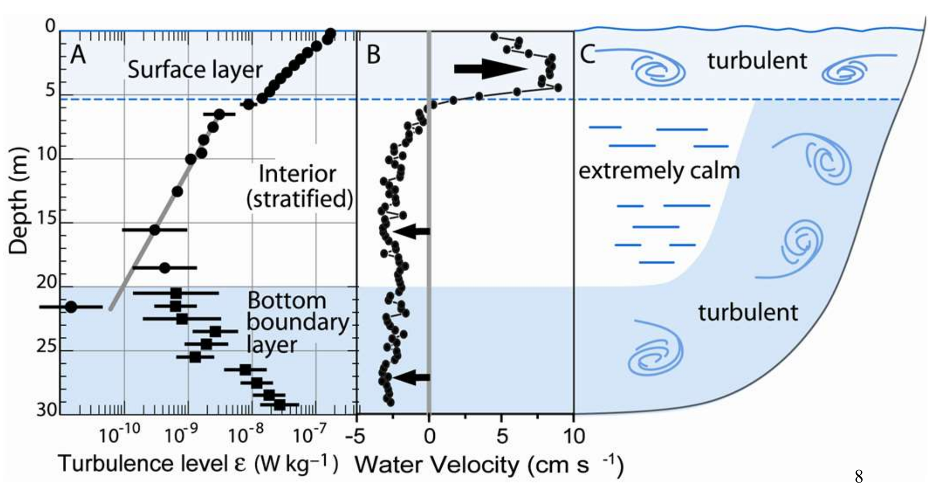
# Surface and bottom boundary layers: two locations of turbulence and exchange

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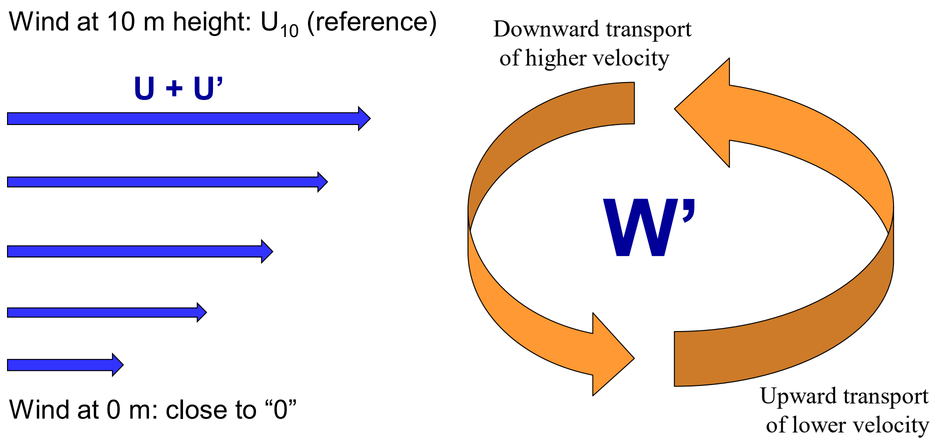
# 1. The physics of the momentum flux




→ **Net effect: $\rho_A$<U'W'> downward**

Vertical (W') turbulent flux of U-component
Horizontal (U') turbulent flux of U-component

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# 1. The force of wind onto water surface

Assumptions: U' (horizontal) proportional to $U_{10}$

W' (vertical) proportional to $U_{10}$

- Momentum flux = force/area [$Nm^{-2}$] = $\rho_A$<U'W'>
- Surface stress (force/area) [$Nm^{-2}$] = $C_{10}\ \rho_A (U_{10})^2$
- Factor $C_{10}$ to fix the "equation" (non-dim)

**Constant stress across air-water interface water**

$$\tau = \rho_{air}\ \overline{U'W'} = \rho_w\ \overline{u'w'}$$

$$\tau = \rho_{air}\ C_{10}\ U_{10}^2$$

General for turbulence
! Friction force ~ $U^2$ !

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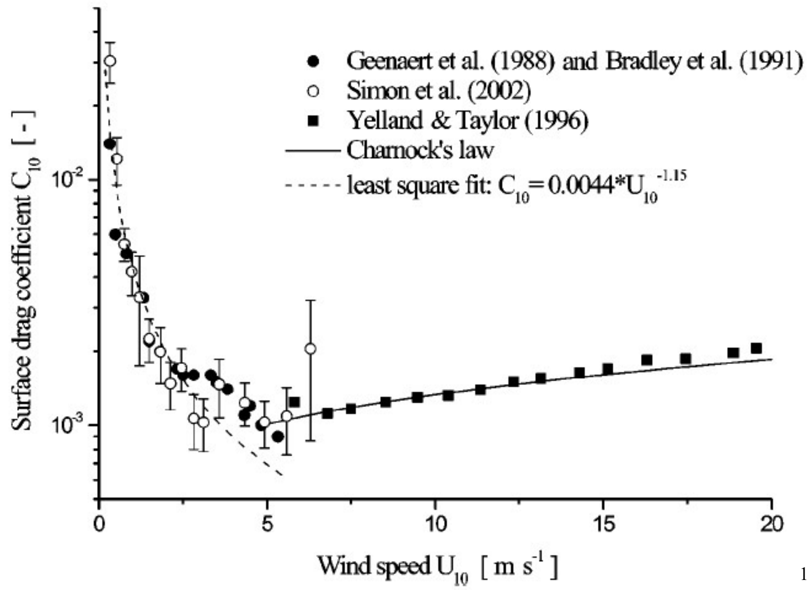
## 1. Drag coefficient as f($U_{10}$)

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## 1. The force of wind onto water surface

$$\tau = \rho_{air} \overline{U'W'} = \rho_w \overline{u'w'}$$

$$\tau = \rho_{air} C_{10} U_{10}^2$$

*Definition* $u_*$ *and* $w_*$

$$\tau = \rho_w u_*^2 = \rho_{air} w_*^2$$

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## 2. Friction velocity = shear velocity (drag velocity)

**Turbulent velocity fluctuations in the LOG-layer**

**Def:** $w_*^2 = \langle U'W' \rangle = \tau / \rho_A$

**Def:** $u_*^2 = \langle u'w' \rangle = \tau / \rho_W$

Sometimes in text / papers sloppily:
$(w_*)^2$ or $w_*$ addressed as shear stress

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## In class exercice:

**1) Friction velocity**

Wind with a velocity of $U_{10}$ = 5 m s-1 blows on a lake. $C_{10} = 1 \cdot 10^{-3}$ $\rho_{air} = 1.25$ kg $m^{-3}$

1) What is the shear stress on the water?
2) Calculate the shear velocities in the air and in the water?
3) What is the ratio between these two velocities?

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## Three boundary layers

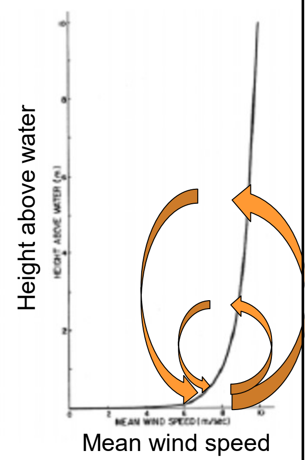
| | |
|---|---|
| Air | |
| DBL | 1mm |
| VBL | 1 cm |
| LOG | a few m |
| Interior | 10s to 100s of m |
| LOG | a few m |
| VBL | 1cm |
| DBL | 1mm |
| Sediment | |

## 3. The Law of the Wall: LOG layer – air side

Prandtl assumption:
$K_m = k\, w_*\, h,$

with k =0.41= von Kármán constant

**eddy sizes and diffusivity increase with distance h to the wall**

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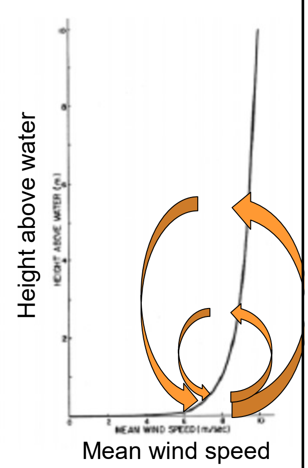
## 3. The Law of the Wall: LOG layer – air side

$$\tau = \rho_{air}\, \overline{U'W'}$$
$$= \rho_{air}\, K_m\, \partial U / \partial z$$

Prandtl: $K_m = k\, w_*\, h,$ *with k =0.41*

**(7.2.5)** $$\frac{\partial U}{\partial h} = \frac{\sqrt{\tau / \rho_{air}}}{k\, h}$$

**(7.2.6)** $$U(z) = U_0 + \frac{w_*}{k} \ln(\frac{h}{h_0})$$




$U_0$= surface roughness = velocity at the air-water interface
H0 = height where velocity reach 0

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# 3. The Law of the Wall: LOG layer – water side

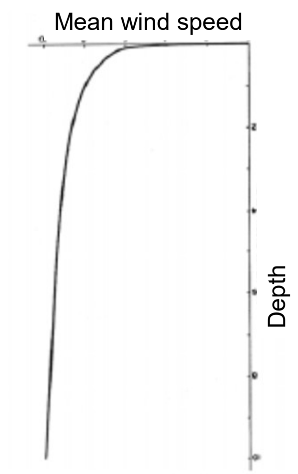
Similar to air side: h → z

Prandtl: $K_m = k\, u_* \, z$

(7.2.8) $$\frac{\partial u}{\partial z} = -\frac{\sqrt{\tau_{SBL}/\rho}}{k\, z}$$

(7.2.9) $$u(z) = U_0 - \frac{u_*}{k}\ln\left(\frac{z}{z_0}\right)$$

(7.2.10) $$u_* = \sqrt{\tau/\rho}$$

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# 3.The Law of the Wall: LOG layer – water side

**Turbulence in LOG-layer**

(7.2.11) $$J_R = -\overline{u'w'}\frac{\partial u}{\partial z} = -(\tau_{SBL}/\rho)\frac{\partial u}{\partial z}$$

(7.2.12) $$J_R = u_*^2 \frac{\sqrt{\tau_{SBL}/\rho}}{k\, z} = u_*^2 \frac{u_*}{k\, z} = \frac{u_*^3}{k\, z}$$

→ The rate of production of turbulent kinetic energy $J_R$ is :

- proportional to the shear velocity $u^3_*$
- inversely proportional to the distance to the interface z

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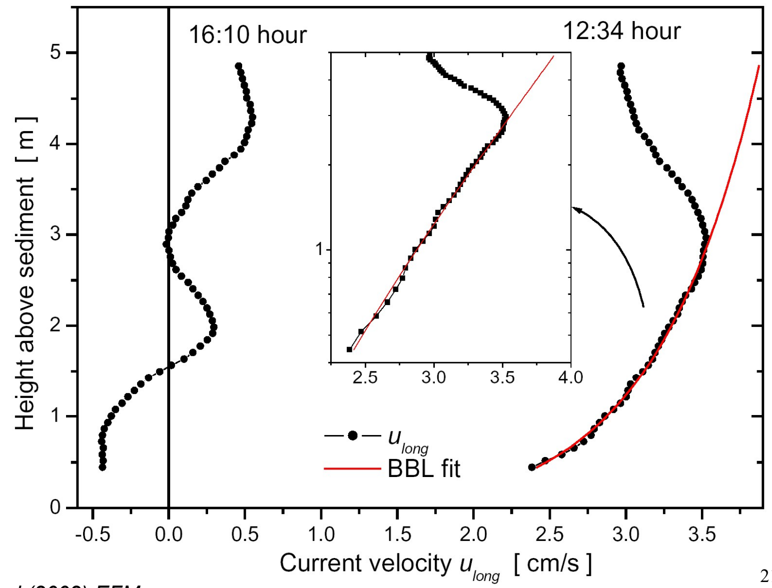
## 3. Near-bottom current profile – Law-of-the-Wall






*Lorke et al (2002) EFM*

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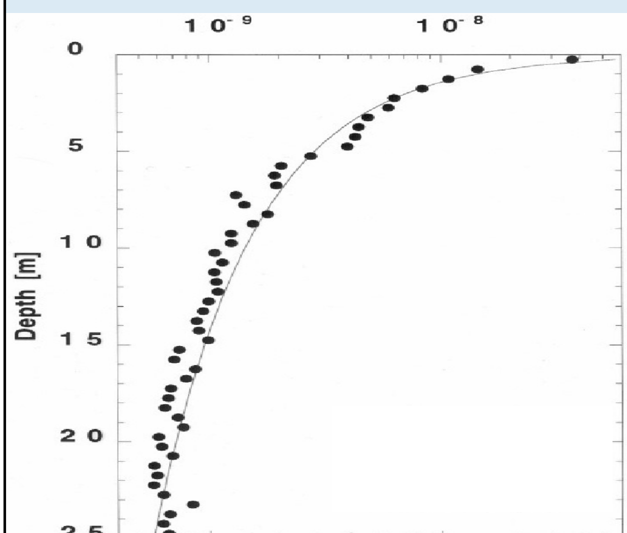
## 3. SBL turbulence
## Energy dissipation $\varepsilon$ = $u_*^3$/kz [W/kg]




$$J_R = \overline{u'w'} \; \partial u / \partial z$$

$$\varepsilon = J_R = u_*^3 \; / kz$$



*Wüest and Lorke 2003, Stips et al. 2005*

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# 4. Viscous boundary layer

The viscous forces dominate the resistance to momentum transfer

The horizontal flow becomes laminar

(7.2.15) $$\tau_{SBL} = \rho\, \nu \frac{\partial u}{\partial z}$$

$$\frac{\partial u}{\partial z} = \frac{\tau_{SBL}}{\rho\, \nu} = \frac{u_*^2}{\nu}$$

(7.2.16) $\delta\nu \approx 10\ \nu/u^*$

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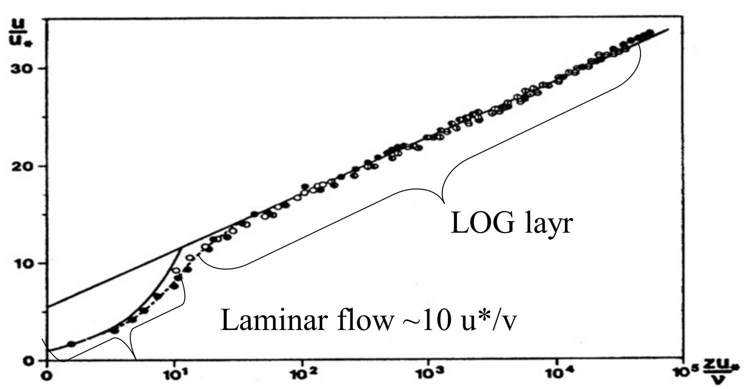
# 4. Viscous boundary layer (non-dimensional axis)




Heights in logarythmic scale $zu_*/\upsilon$

Figure 7.3.1

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# 4. Viscous boundary layer

Turbulent kinetic energy dissipation:

(7.3.9) $$\varepsilon_v = \frac{\tau_{BBL}}{\rho}\frac{\partial u}{\partial z} = \frac{u_*^4}{\nu} \qquad [\mathrm{W\ kg^{-1}}]$$

Total dissipation in the viscous layer:

(7.3.10) $$P_{vdiss} = \rho\ \varepsilon_v\ \delta_v \approx 10\ \rho\ u_*^3 \qquad [\mathrm{W\ m^{-3}}]$$

In the BBL:

- 40% of the energy is dissipated in the VBL: laminar (no mixing)
- 60 % of the energy is dissipated in the LOG-layer: turbulent (causing mixing)

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# In class exercice:

**2) Get used to boundary layer quantities**

Wind blows with a velocity of $U_{10}$ = 7 m $s^{-1}$ over a water body and induces a mechanical energy flux ($P_{10}$) from the atmosphere towards the water surface at 10 m height: $$P_{10} = \tau * U_{10} = \rho_{air} * C_{10} * U_{10}^3$$

a. Calculate the surface stress $\tau$ [N $m^{-2}$] that the wind is exercising on the water
b. Calculate the mechanical energy flux ($P_{10}$, [W $m^{-2}$]) from the atmosphere towards the water surface at 10 m height.
c. Calculate the friction velocity w* (in the air BL) and u* (in the water BL).
d. Calculate the VBL thickness on the airside ($\delta$air) and on the waterside ($\delta$water) of the air-water interface. Which one is broader?

Kinematic viscosity: $\nu_{water}$ (5 ºC) =1.52·$10^{-6}$ $m^2$ $s^{-1}$; $\nu_{air}$ (15 ºC) = 14.5·$10^{-6}$ $m^2$ $s^{-1}$

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# In class exercice:

**2) Get used to boundary layer quantities**

Wind blows with a velocity of $U_{10}$ = 7 m $s^{-1}$ over a water body and induces a mechanical energy flux ($P_{10}$) from the atmosphere towards the water surface at 10 m height:

$$P_{10} = \tau * U_{10} = \rho_{air} * C_{10} * U_{10}^3$$

d. Calculate the dissipation in air at 10 m above surface and in water in 1 m depth.
e. How do you calculate the total dissipation between VBL and 10 m in the atmosphere and between VBL and 1 m in the water?

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# 5. The diffusive boundary layer

**(7.2.17)** $$Flux = -D_C \frac{\partial C}{\partial z} \approx D_C \frac{\Delta C}{\delta_{DBL}} = \frac{D_C}{\delta_{DBL}} \Delta C = V_{tot}\ \Delta C$$

Two interpretations:

(i) Molecular diffusion along **gradient $D_C/\delta_{DBL}$**

**(ii) Driving force $D_C$** and exchange velocity $V_{tot}$

**Case air-water:** $V_{tot}$ = gas exchange velocity or piston velocity (see chapter 6)

**Case sediment-water**: $V_{tot}$ = mass exchange velocity (any dissolved substance: gas, ions, DIC, organics, etc)

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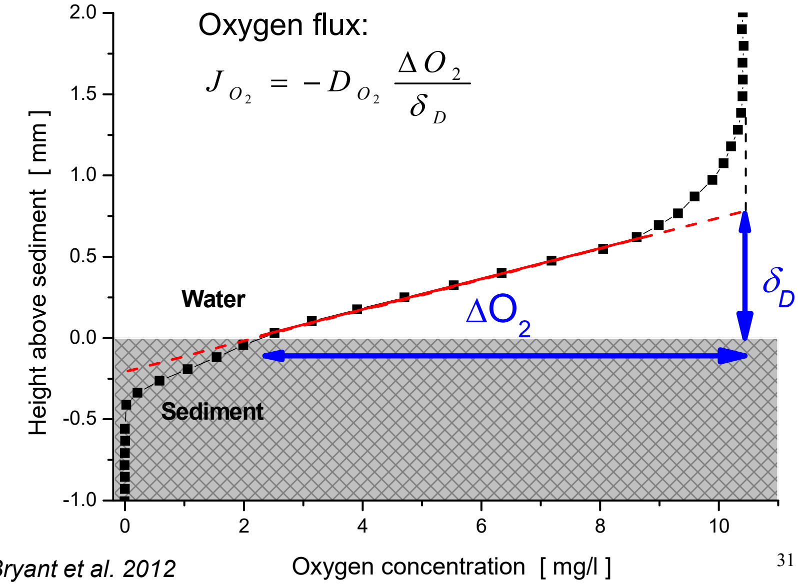
## 5. The diffusive boundary layer




*Bryant et al. 2012*

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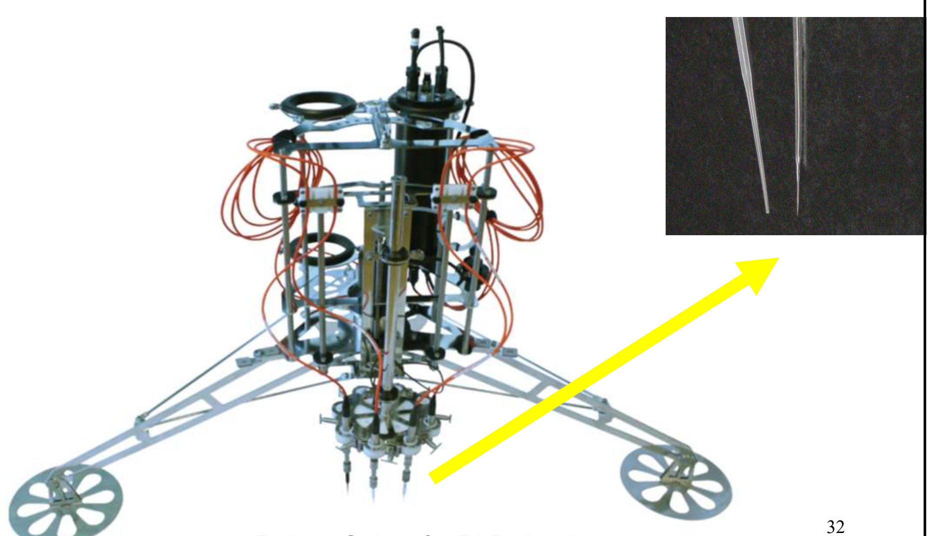
## 5. In-situ robot for 100 Micrometer oxygen profiling resolution



*Robert Schwefel PhD thesis*

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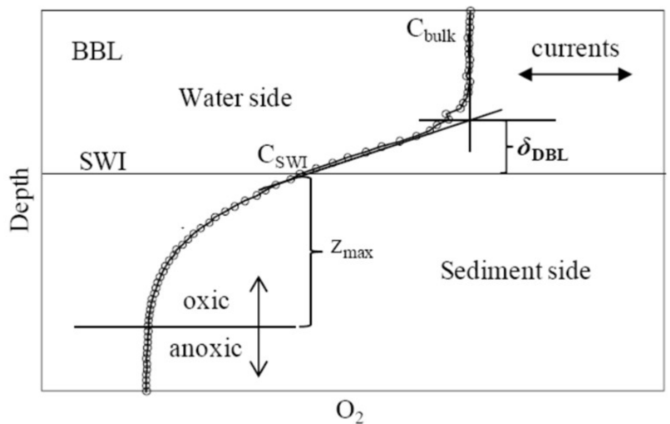
## 5. Short-term → fast reaction




Content of $O_2$ in Sediment = 0.032 mmol $m^{-2}$
Flux into sediment 5 mmol $m^{-2}$ $d^{-1}$

→ Residence time = 9 minutes

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## Three boundary layers – review relative to level of diffusivity K

| | | |
|---|---|---|
| $K >> \nu$ | → LOG layer | |
| $K = \nu$ | → distance $\delta_\nu$ | Limit viscous BL |
| $K < \nu$ | → viscous BL | |
| $K = D_C$ | → distance $\delta_{DBL}$ | Limit diffusive BL |
| $K < D_C$ | → diffusive BL | |

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# Three boundary layers

| | |
|---|---|
| Air | |
| DBL | 1mm |
| VBL | 1 cm |
| LOG | a few m |
| Interior | 10s to 100s of m |
| LOG | a few m |
| VBL | 1cm |
| DBL | 1mm |
| Sediment | |

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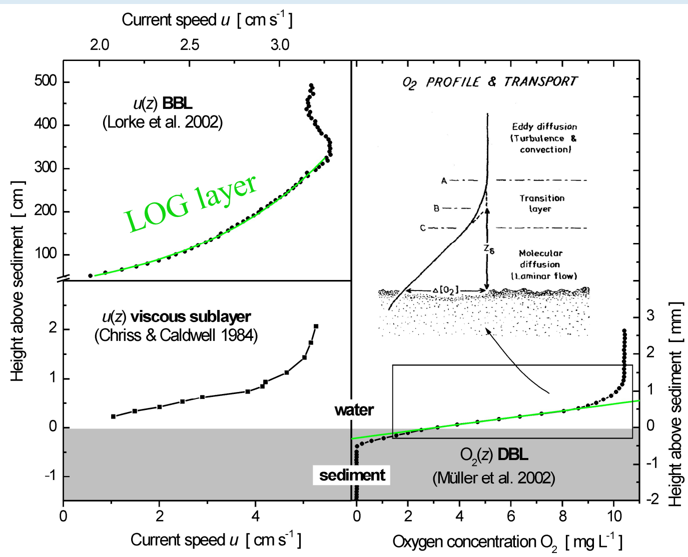
# Summary: Three levels (sizes) of BBL

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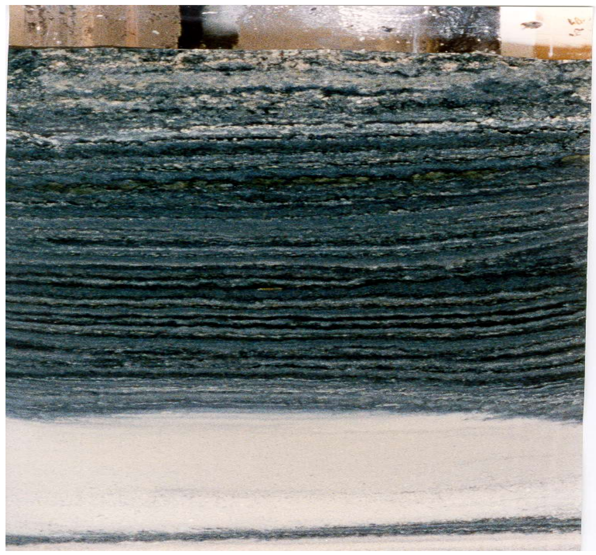
## Ecological relevance of the diffusive boundary layer



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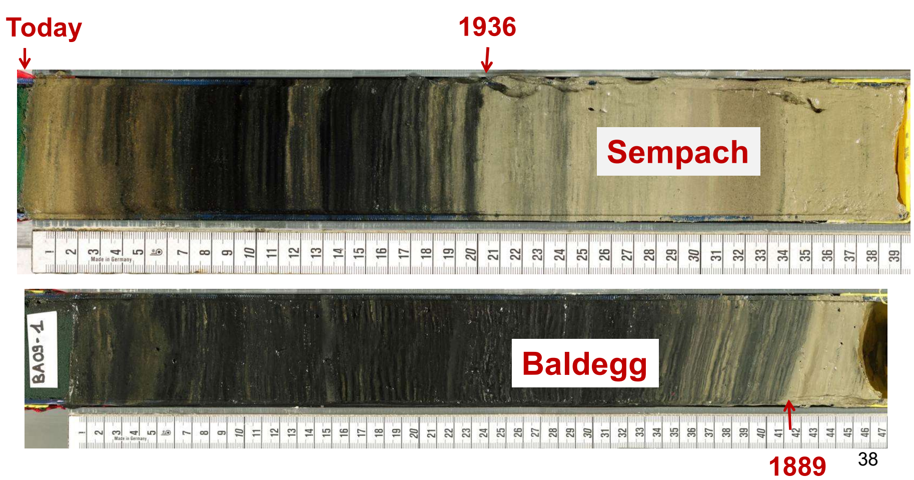
## Effect of aeration on sediment?

Higher $O_2$ level → more flux of $O_2$ into the sediment → more depletion → less C in sediment (see color)

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## Take home message

1. Wind forcing: surface shear stress $\tau = \rho_{air} \overline{U'W'} = \rho_w \overline{u'w'}$
   $\tau = \rho_{air} C_{10} U_{10}^2$
2. Friction/shear velocity: $w_*^2 = \langle U'W' \rangle = \tau / \rho_A$ $u_*^2 = \langle u'w' \rangle = \tau / \rho_W$
3. Three main layers: LOG (~m) >> VBL (~cm) > DBL (~mm) > interface
   - Logarythmic BL: turbulent, K >> $\upsilon$, dissipation $\varepsilon_\upsilon = u_*^3/kz$
     $$u(z) = U_0 - \frac{u_*}{k} \ln(\frac{z}{z_0})$$
   - Viscous BL: laminar, K ≤ $\upsilon$, $\delta_\upsilon \approx 10\ \upsilon/u_*$ ; dissipation $\varepsilon_\upsilon = u_*^4/\upsilon$
   - Diffusive BL: laminar, K ≤ $D_C$, Flux = $-D_C/\delta_{DBL}\ \Delta C$
4. Ecological relevance: many key processes accross boundary layers:
   - Primary production, light, heat and gas transfers, mass and solutes transfers

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## In class exercice:

**3) Ecological relevance of boundary layers**

a. Since the 1980s, the three deep-waters of three lakes on the the Swiss Plateau (Lakes Sempach, Baldegg and Hallwil) are artificially oxygenated and artificially mixed during wintertime. Therefore, the deep-water oxygen concentrations are now higher than before the 1980s. Today the oxygen consumption in the deep-water is higher than before the 1980s, despite that water quality has improved? Why?
b. Consider a shallow lake and a deep ocean basin with both the same annual primary production. In which of the two sediments do you expect higher / lower organic carbon content?

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